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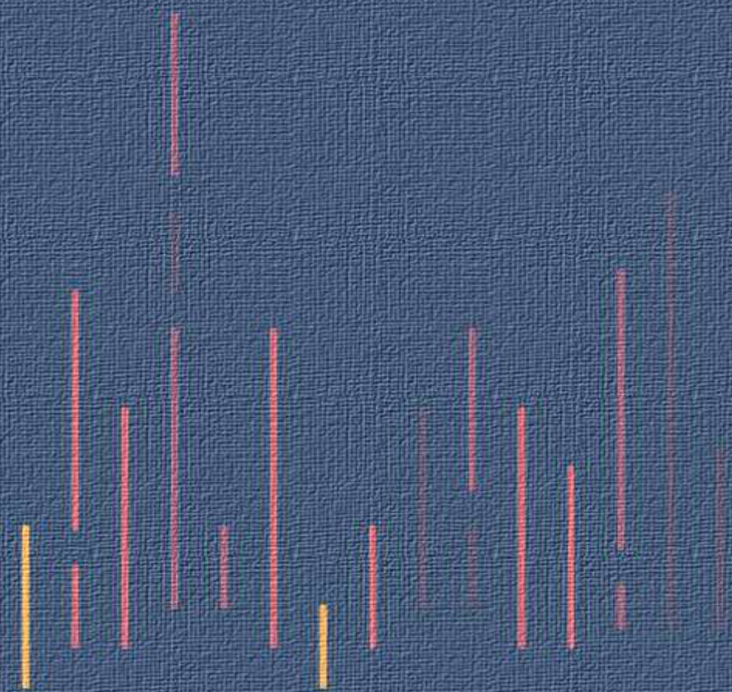

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“Экспериментал тадқиқотлар” илмий-амалий журнали 6 та халқаро маълумотлар базаларида индексланган бўлиб, жорий йил учун UIF 2023 = 7.4 “импакт-фактор” кўрсатгичига эга. Ўзбекистон Республикаси Олий таълим, фан ва инновациялар вазирлиги ҳузуридаги Олий аттестация комиссиясининг 2023 йил 24 июлдаги 01-02/1199-сонли хатига мувофиқ ушбу журналда чоп этилган мақолалар хорижий мақолалар сифатида тан олинади.

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ЖУРНАЛ ЭКСПЕРИМЕНТАЛЬНЫХ ИССЛЕДОВАНИЙ | JOURNAL OF EXPERIMENTAL STUDIES

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PURSUIT DIFFERENTIAL GAMES WITH CONSTRAINTS OF GRONWALL TYPE

ANNOTATION

In this article, a differential game of pursuit has been studied when controls of conflicting objects belong to the classes of Gronwall type constraints. Here a parallel approach strategy will be proposed for pursuer and by virtue of this, a solution of pursuit problem will be given. General information about simple differential games is presented throughout the article. It contains information about the basic concepts of the theory of differential games such as strategy, problems of chasing and escaping in integral, geometric, integral-geometric and other bounded differential games. In addition, in order to demonstrate the processes of solving the problem in classical games, the solution of a problem with an integral limit is shown in full.

Keywords: Differential game, Gronwall's inequalities, players, Gronwall constraints, pursuit, strategy,chase,escape.

CHEKLOVLAR BILAN DIFFERENTSIAL O'YINLARNI TA'QIB QILISHNING GRONUOLL TURI

ANNOTATSIYA

Mazkur maqolada qarama-qarshi obyektlarni boshqarish elementlari Gronuoll tipidagi chegaralanishlar sinfiga kirganda, differensial quvish masalasi o'rganildi. Bu yerda quvuvchi uchun parallel yondashuv strategiyasi taklif etiladi va shu sababli quvish muammosini hal etish beriladi. Oddiy differentsial o'yinlar to'g'risidagi umumiy ma'lumot butun maqolada taqdim etilgan. U differentsial o'yinlar teoriyasining strategiyasi, integral, geometrik, integral-geometrik va boshqa chegaralangan differentsial o'yinlarda quvish va qochish masalalari kabi asosiy tushunchalari haqida ma'lumotni o'z ichiga oladi. Bundan tashqari, klassik o'yinlarda vazifani hal etish jarayonlarini namoyish etish uchun vazifani integral –geometrik chegara bilan hal etish to'liq ko'rsatilgan.

Kalit so'zlar: differensial o'yin, Gronwall tengsizliklari, o'yinchilar, Gronwall chegaralari, ta'qib qilish, strategiya, quvish, qochish.

ПОГОНЯ ЗА ДИФФЕРЕНЦИАЛЬНЫМИ ИГРАМИ С ОГРАНИЧЕНИЯМИ ГРОНУОЛЛА ТИП ИЗГОТОВЛЕНИЯ

АННОТАЦИЯ

В этой статье была изучена дифференциальная игра преследования, когда элементы управления конфликтующими объектами относятся к классам ограничений типа Гронуолла. Здесь будет предложена стратегия параллельного подхода для преследователя, и в силу этого будет дано решение проблемы преследования. Общая информация о простых дифференциальных играх представлена во всей статье. Он содержит информацию об основных понятиях теории дифференциальных игр, таких как стратегия, проблемы преследования и побега в интегральных, геометрических, интегрально-геометрических и других ограниченных дифференциальных играх. Кроме того, для демонстрации процессов решения задачи в классических играх решение задачи с интегральным пределом показано в полном объёме.

Ключевые слова: дифференциальная игра, неравенства Gronwall, игроки, ограничения Gronwall, преследование, стратегия, погоня, побег.

1. Introduction

Differential games are a complex mathematical concept due to the need to take into account factors such as dynamism, controllability, conflict, optimality, informativeness. Such factors require the development of special approaches to the theory of differential games. Such an approach depends primarily on solving chase-escape problems, which is an important branch of differential game theory. Differential games are one of the important branches of mathematical management theory. The theory of differential games develops the ideas and methods of optimal control theory. The basic mathematical tool of the theory of differential games is the theory of ordinary differential equations. With the help of differential equations, a number of important laws of physics, mechanics, chemistry, biology, astronomy, economics, management theory are written and researched.

2. Materials and methods.

In the scientific results of most chase-escape problems of the theory of differential games, the control functions are selected from the classes limited by the limit of assignment.The basic concepts of differential game theory depend on sets, the theory of mathematical analysis, the theory of differential equations, the theory of optimal control, and the basic concepts of the theory of linear algebra. In the theory of differential games, the concept of strategy occupies a central place. In the theory of differential games, a definition is given based on the nature of the problem.

3. Results and discussion.

Suppose that in space R^n the chasing P and escaping E objects are given, so that their equations of motion are given based on the following differential equations, respectively

$$P: \dot{x} = u, \quad x(0) = x_0 \qquad (1)$$

$$E: \dot{y} = v, \quad y(0) = y_0 \qquad (2)$$

here $x, y, u, v \in R^n$, $n \geq 2$, $a \neq 0$. Here u -is the velocity vector of the inverter whose change over time $u(\cdot):[0,\infty) \to R^n$ is selected from the class of functions measured in Lebesgue measure and given the followingGronwall-type bounding (Gr-bounding)

$$|u(t)|^2 \le \rho^2 + 2k_1 \int_0^t |u(s)|^2 ds, \quad \text{almost all} \quad t \ge 0, \qquad (3)$$

here $\rho > 0$, $k_1 > 0$. Let's define this (3) constraint as the set of all measurable functions U_{Gr} that satisfy it.

Similarly, the velocity vector v of a fluid $v(\cdot):[0,+\infty) \to R^n$ is selected as a Lebesgue measure of its change over time and given the following Gronwall-type constraint:

$$|v(t)|^2 \le \sigma^2 + 2k_2 \int_0^t |v(s)|^2 ds \qquad \text{almost all,} \qquad (4)$$

here $\sigma \ge 0$, $k_2 > 0$. Here, we define (4) as the set of all measurable functions V_{Gr} that satisfy the Grenouille constraint.

From the equations (1)-(2) and for the pairs $(x_0, u(\cdot))$ va $(y_0, v(\cdot))$, the following trajectories of movement of the chasing and escaping objects are derived respectively:

$$x(t) = x_0 + \int_0^t u(s)ds, \qquad y(t) = y_0 + \int_0^t v(s)ds$$

The purpose of a pushing object is to capture an escaping object in a $t^* > 0$ finite amount of time, i.e.

$$x(t^*) = y(t^*)$$

it's about equality. The objective of the fugitive object is to avoid encountering the pursuing object, to make it accessible to all $t \ge 0$, or to delay the encounter $x(t) \ne y(t)$ as long as possible.

Lemma. (Gronwall's lemma). If this

$$|\omega(t)|^2 \le \alpha^2 + 2l \int_0^t |\omega(s)|^2 ds,$$

the inequality is real, then the relationship $|\omega(t)| \le \alpha e^{lt}$ is always real in the interval of $t \ge 0$. Here $\omega(t)$ is the measurable function and α, l are the negative numbers.

Definition-1. In the (1)-(4) differential game

$$u_{Gr}(t,v) = v - \lambda_{Gr}(t,v)\xi_0 \qquad (5)$$

the parallel driving strategy of the driving object is called the strategiya [3]) where $\lambda(t,v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + \rho^2 e^{2k_1 t} - |v|^2}$, $\xi_0 = z_0 / |z_0|$ and $< v, \xi_0 >$ - R^n is a scalar product of vectors in space v and ξ_0.

Property-1. If $\rho \ge \sigma$, $k_1 > k_2$ the inequality is true, then the function $\lambda_{Gr}(t,v)$ is always defined in $t \ge 0$ and nonnegative for all $|v| \le \sigma e^{kt}$ controls.

Property -2. The function $\lambda_{Gr}(t,v)$ is limited to all $|v| \le \sigma e^{kt}$ controls , and the following relationship is performed at $t \ge 0$

$$\rho e^{k_1 t} - |v| \le \lambda_{Gr}(t,v) \le \rho e^{k_1 t} + |v|.$$

Theorem. If the following condition holds, if $\rho > \sigma$, $k_1 \ge k_2$ is true, then the strategy in the (1)-(4) differential game (6) is winning for the pursuer in the $[0, T_{Gr}]$ time interval, and the following T_{Gr} is chosen as the smallest positive solution to the equation

$$\rho k_2 e^{k_1 t} - \sigma k_1 e^{k_2 t} = \rho k_2 - \sigma k_1 + k_1 k_2 \, | z_0 | .$$

Proof. Suppose that if the runner chooses voluntary $v(\cdot) \in V_{Gr}$ control, and the pursuer chooses strategy (6), then we obtain the following Caratheodory equation based on (1) and (2):

$$\begin{cases} \dot z(t) = -\lambda(\mathrm{t}, v(\mathrm{t}))\xi_0 \\ z(0) = z_0. \end{cases}$$

From this , we can derive the following solution given the initial conditions:

$$z(t) = z_0 - \int_0^t \lambda(s, v(s))\xi_0 ds$$

or

$$z(t) = z_0 - \int_0^t \left(\langle v(s), \xi_0 \rangle + \sqrt{\langle v(s), \xi_0 \rangle^2 + \rho^2 e^{2k_1 s} - |v(s)|^2} \right) \xi_0 ds.$$

using the above equation, we can derive the following equation:

$$z(t) = z_0 \Lambda(t, v(\cdot)) \qquad (6)$$

here, $\Lambda(t, v(\cdot))$ is equal

$$\Lambda(t, v(\cdot)) = 1 - \frac{1}{|z_0|} \int_0^t \lambda(s, v(s)) ds .$$

We're going to study $\Lambda(t, v(\cdot))$ the approximation function over time t. To do this , we use Definition-1 and Property-2 to produce the following equation:

$$\Lambda(t, v(\cdot)) = 1 - \frac{1}{|z_0|} \int_0^t \lambda(s, v(s)) ds =$$

$$= 1 - \frac{1}{|z_0|} \int_0^t \left(\langle v(s), \xi_0 \rangle + \sqrt{\langle v(s), \xi_0 \rangle^2 + \rho^2 e^{2k_1 s} - |v(s)|^2} \right) ds \le$$

$$\le 1-\frac{1}{|z_0|}\int_0^t\left(\rho e^{k_1 s}-\sigma e^{k_2 s}\right)ds=\Lambda(t)$$

here

$$\Lambda(t)=1-\frac{1}{|z_0|}\left(\frac{\rho}{k_1}(e^{k_1 t}-1)-\frac{\sigma}{k_2}(e^{k_2 t}-1)\right).$$

$\Lambda(t)=0$ equals the following:

$$\rho k_2 e^{k_1 t}-\sigma k_1 e^{k_2 t}=\rho k_2-\sigma k_1+k_1 k_2|z_0|.$$

The function $\Lambda(t)$ is monotonically decreasing and equal to the $\Lambda(T_{Gr})=0$ interval $[0,T_{Gr}]$ according to the theorem. So, we have a time interval$t^*, 0\le t^*\le T_{Gr}$, where $\Lambda_{Gr}\left(t^*,v(\cdot)\right)=0$. Based on the equality (6), we get $z(t^*)=0$.

The theorem is proved.

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